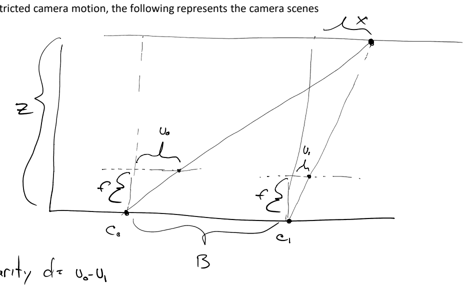
Assignment #4

Tuesday, August 29, 2017 5:12 PM

1) Assuming restricted camera motion, the following represents the camera scenes



disparity $d = u_0 - u_1$

By similar triangles: $\frac{u_0}{f} = \frac{B+x}{Z}$ & $\frac{u_1}{f} = \frac{x}{Z}$

Thus $\frac{u_0 - u_1}{f} = \frac{B + x - x}{Z}$

$\therefore \quad Z = f\frac{B}{d}$

2)

In this example, we simplify the computation of the energy functional using the following recurrence:

where $E_{data} = [I_0(x+d, y) - I_1(x,y)]^2$

$E_{smooth} = \lambda$, constant cost for discontinuity

$$c(i,j) = \begin{cases} c(i-1, j-1) + E_{data}, & \text{case } 1 \\ c(i-1, j) + E_{smooth}, & \text{case } 2 \\ c(i, j-1) + E_{smooth}, & \text{case } 3 \end{cases}$$

$c(0,j) = c(0,j-1) + E_{smooth}$

$c(i,0) = c(i-1,0) + E_{smooth}$

I_0: | 4 | 4 | 5 | 5 | 8 |

I_1: | 6 | 6 | 6 | 7 | 10 |

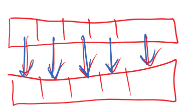
	X	4	4	5	5	8
X	0	λ	2λ	3λ	4λ	5λ
6	λ	4	12	17	25	33
6	2λ	12	8	13	18	26
6	3λ	20	16	9	14	22
7	4λ	28	24	17	13	15
10	5λ	36	32	25	21	17

$E_{data} = [I(j,y) - I(i,y)]^2$

Assume

$E_{smooth} = [\lambda] = 8$

Answers may vary based on recurrence and penalty.



d: | 0 | 0 | 0 | 0 | 0 |

You can repeat this process for rows 2 and 3.

3) We will cover graph-based methods in more detail in upcoming lectures. Here I give a brief explanation for a global solution.

A global solution can be formulated by mapping this problem to a 3-D graph. (See Roy et al). (Note also, that Boykov et al provide a local solution with a fast approximation.)

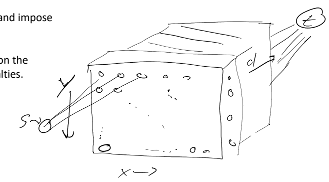
Each node corresponds to a triple (x,y,d) where (x,y) is pixel coordinate and d is a potential disparity label.

A source s and a sink t are connected to the 3-d graph. The source connected to all nodes with a min disparity and the sink connected to all nodes associated with a max disparity.

Note a min cut will separate the source and sink, and impose A disparity map.

The edge capacities are intuitively defined based on the Matching costs and discontinuity (occlusion) penalties.

Intuition: Good matches will have a low cost, thus low capacity and will be saturated by a Maxflow. Similarly this edge will be selected In a min cut.



4) Here we assume the first operand is "slid" across the second. NOTE: Results will vary based on boundary conditions and based on which operand is "slid"

A) $I * K = G$ $[5\ -3\ -2\ 2\ 3]$ I

$*$ $-1\ 0\ 1$

$G = [-3\ -2\ 5\ 5\ -7\ -3\ 5]$

B) $K * I = G$ $[1\ 0\ -1]$ K

$*$ $[3\ 2\ -2\ -3\ 5]$

$G = [-3\ -2\ 5\ 5\ -7\ -3\ 5]$

Note: $I * K = K * I$

C) $I \otimes K = G$ $[3\ 2\ -2\ -3\ 5]$

$\otimes$ $[-1\ 0\ 1]$

$G = [-5\ 4\ 7\ -5\ -5\ 2\ 3]$

D) $K \otimes I = G$ $[-1\ 0\ 1]$

$[3\ 2\ -2\ -3\ 5]$

$G = [3\ 2\ -5\ -5\ 7\ 4\ -5]$

5) $K = [0\ 1\ -1]$

$\nabla_x I = K * I$

Other kernels are common. Note however this one explicitly represents A finite difference as defined in class.

6) Here we extend beyond the boundries and provide a result anywhere there is a non-zero result.

A) $I = \begin{bmatrix} 1 & 2 & -2 & 3 & 10 \\ 3 & 2 & 0 & 3 & 5 \\ 5 & 4 & 0 & 2 & 1 \end{bmatrix}$

$K = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

$I \otimes K = \begin{bmatrix} -1 & -2 & 1 & -2 & -5 & 4 & 5 \\ -5 & -1 & 9 & 1 & 5 & 12 & 3 \\ -9 & 9 & 17 & 3 & 6 & 4 & -4 \\ 5 & 25 & 1 & -5 & 7 & 0 & 3 \\ 10 & 3 & -12 & -1 & 3 & -2 & -1 \end{bmatrix}$

B) $\nabla^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} = L$

$K = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$

$\nabla^2 K = L \otimes K = \begin{bmatrix} 0 & -1 & 0 & 1 & 0 \\ -1 & 4 & 2 & -4 & 1 \\ 0 & 2 & -8 & 2 & 0 \end{bmatrix}$

b)
$$\nabla^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} = L$$

$$K = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\nabla^2 K = L \otimes K = \begin{bmatrix} 0 & -1 & 0 & 1 & 0 \\ -1 & 4 & 2 & -4 & 1 \\ 0 & 2 & -8 & 2 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

c)
$$I \otimes \nabla^2 K = \begin{bmatrix} 0 & -1 & -2 & 1 & -2 & -5 & 4 & 5 & 0 \\ -1 & -3 & 7 & 1 & 5 & 27 & -4 & -13 & 5 \\ -5 & 9 & 15 & -18 & 11 & -6 & 32 & 1 & 3 \\ -9 & 45 & -6 & -46 & 7 & -5 & -2 & 20 & -4 \\ 5 & 4 & -24 & 19 & 30 & -24 & 6 & 7 & -3 \\ 10 & -32 & 9 & 5 & 1 & -10 & -8 & 10 & -1 & -1 \\ 0 & 10 & 3 & -12 & -1 & 3 & -2 & -1 & 0 \end{bmatrix}$$

D)
$$\nabla_x (I \otimes K) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 3 & -3 & -3 & 9 & 1 & -5 \\ 0 & -5 & 4 & 10 & -8 & 4 & 7 & -9 & -3 \\ 0 & -9 & 18 & 8 & -4 & 3 & -2 & -8 & 4 \\ 0 & 5 & 18 & -22 & -6 & 12 & -7 & -3 & 3 \\ 0 & 10 & -7 & -15 & 11 & 4 & -5 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\nabla_x = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\nabla_y = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\nabla_y (I \otimes K) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 1 & -2 & -5 & 4 & 5 & 0 \\ 0 & -4 & 1 & 8 & 3 & 10 & 8 & -2 & 0 \\ 0 & -4 & 10 & 8 & 2 & 1 & -8 & -7 & 0 \\ 0 & 14 & 14 & -16 & -8 & 1 & -4 & 1 & 0 \\ 0 & 5 & -20 & -13 & 4 & -4 & -2 & 2 & 0 \\ 0 & -10 & -3 & 12 & 1 & -3 & 2 & 1 & 0 \end{bmatrix}$$

7) Let $G(i,j) = \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} H(u,v) \cdot F(i+u, j+v)$

Let f_1 & f_2 be functions and $f_3 = f_2 + f_1$. Let h be function.

$$h \otimes (f_1 + f_2) = h \otimes f_3 = g(i,j) = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} h(u,v)\, f_3(i+u, j+v)$$

$$= \sum\sum h(u,v)\left[f_1(i+u, j+v) + f_2(i+u, j+v)\right]$$

$$= \sum\sum h(u,v) f_1(i+u, j+v) + h(u,v) f(i+u, j+v)$$

$$= \sum\sum h(u,v) f_1(i+u, j+v) + \sum\sum h(u,v) f(i+u, j+v)$$

$$= h \otimes f_1 + h \otimes f_2$$

$\blacksquare$