Computer Vision **Name:** ____________________

Prof. Bolton **Net ID:** ____________________

Assignment 4

This assignment contains 2 pages (including this cover page) and 7 questions. Total of points is 100.

Conditions: Work in groups of two or less.

Include your name and Net ID. Follow submission instructions as indicated herein.

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1. (10 points) Read Bolles Section 1. In the paper, stereo imagery is assumed to be collected from restricted camera motion, or more generally, the images are assumed to be rectified such that corresponding epipolar lines are horizontal and colinear. Using the discussion and geometry of the cameras and scene, derive the following equation for Z, which is the depth, in term of f, the focal length, B, the baseline, and d, the disparity, using similar triangles. To support your derivation, create a geometric illustration of the camera scene. State all assumptions used in the derivation. In your supporting illustration, **clearly identify**the corresponding triangles, the focal length, the baseline, and the disparity.

   $Z = f\frac{B}{d}$

2. (20 points) Solve dynamic programming problem. Assume the following two 3 x 5 stereo images have been rectified. Thus epipolar lines are horizontal and corresponding epipolar lines are colinear (same row). Using the following energy functional, as an objective and the dynamic programming solution presented in class, find the optimal disparity map, i.e., the map which characterizes the optimal point correspondence sequence for each row in the images. Please show the resulting disparity map.

   $E_{total}(d) = \sum_{x,y} [I_0(x + d(x, y), y) - I_1(x, y)]^2 - \lambda \sum_{x,y} [(d(x, y) - d(x - 1, y)]^2$, where $\lambda = 1$.

   $$I_0 = \begin{bmatrix} 4 & 4 & 5 & 5 & 8 \\ 0 & 1 & 2 & 3 & 4 \\ 3 & 1 & 0 & 0 & -1 \end{bmatrix}$$

   $$I_1 = \begin{bmatrix} 6 & 6 & 6 & 7 & 10 \\ 2 & 3 & 3 & 4 & 5 \\ 5 & 4 & -7 & 1 & -3 \end{bmatrix}$$

3. (20 points) Find the optimal disparity map for the rectified stereo images below. Use the min-cut approach presented in class [Boykev et al]. Assume reasonable disparity values are known to be limited to 1, 2, or 3. Use the following Energy Functional: $E_{total}(d) = \sum_{x,y} [I_0(x + d(x, y), y) - I_1(x, y)]^2 - \lambda \sum_{x,y} [(d(x, y) - d(x + 1, y)]^2$, where $\lambda = .5$.

$$I_0 = \begin{bmatrix} 1 & 2 & -2 & 3 & 10 \\ 3 & 2 & 0 & 3 & 5 \\ 5 & 4 & 0 & 2 & 1 \end{bmatrix}$$

$$I_1 = \begin{bmatrix} 0 & 1 & 1 & 8 & 10 \\ 2 & 1 & 3 & 4 & 5 \\ 2 & -1 & 1 & 1 & 3 \end{bmatrix}$$

A. Illustrate the initial state of the graph. Describe nodes, labels, edges and the values of edge weights.

B. illustrate the optimal disparity map corresponding to the min cut

4. (10 points) Given the following 1-d image I (signal) and 1-d kernel K (filter)

   I=

$$\begin{bmatrix} 3 & 2 & -2 & -3 & 5 \end{bmatrix}$$

   and Kernel K=

$$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

   compute the following expressions using convolution $\star$ and cross correlation $\bigotimes$. Explain what you have done with respect to *boundary concerns*.

   A. $I \star K$

   B. $K \star I$

   C. $I \bigotimes K$

   D. $K \bigotimes I$

5. (10 points) Identify an appropriate kernel K and write the following gradient operation (in the x-direction) on image I as a cross correlation with that kernel K. That is, find K such that $\nabla_x I = K \star I$

6. (20 points) Given the following Image $I = \begin{bmatrix} 1 & 2 & -2 & 3 & 10 \\ 3 & 2 & 0 & 3 & 5 \\ 5 & 4 & 0 & 2 & 1 \end{bmatrix}$ and Kernel $K = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix}$, compute the following expressions where cross correlation operator is represented as $\bigotimes$, Laplacian operator is represented as ∇^2, and gradient operator is ∇. Explain what you have done with respect to boundary concerns.

   A. $I \bigotimes K$

   B. $\nabla^2 K$

   C. $I \bigotimes \nabla^2 K$

   D. $\nabla(I \bigotimes K)$

7. (10 points) Let h be a kernel, and let f_1 and f_2 be images. Prove whether the cross correlation operator follows the superposition principle (See [Sz] eq 3.5). That is, show $h \bigotimes (f_1 + f_2) = h \bigotimes f_1 + h \bigotimes f_2$.