Assignment 1

Tuesday, August 29, 2017 5:11 PM

1) $\tilde{x}_1 = (2,4,2) = 2(1,2,1)$. Inhomogeneous form is $x_1 = [1,2]^T \in \mathbb{R}^2$

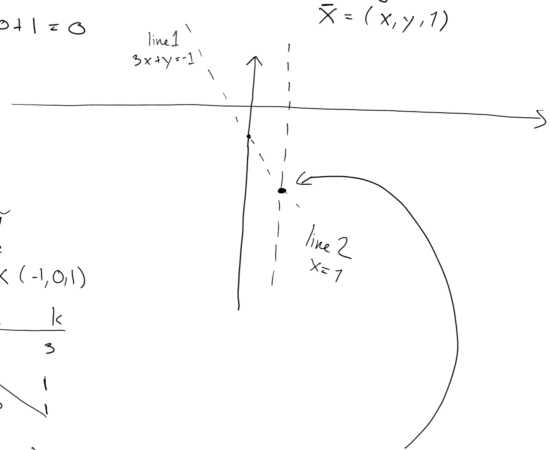
$\tilde{x}_2 = (2,4,1,3) = 3(2/3, 4/3, 1/3, 1)$. Inhomogeneous form $x_2 = [2/3, 4/3, 1/3]^T \in \mathbb{R}^3$

2)

coefficients for 2-d line equation

$\tilde{l}_1 = (3,1,1)$, $\bar{x}\tilde{l}_1 = 3x + y + 1 = 0$ ... Recall $\bar{x}$ is augmented vector $\bar{x} = (x, y, 1)$

$l_2 = (-1,0,1)$, $\bar{x}\tilde{l}_2 = -x + 0 + 1 = 0$



c) Point of intersection $\tilde{x} = \tilde{l}_1 \times \tilde{l}_2$

$= (3,1,1) \times (-1,0,1)$

	i	j	k
i	-3	0	3
j	-1	0	1
k	-1	0	1

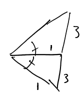
$= (+1, -3, 1)$, thus point of intersection $x_3 = (1,-3)$

d) Observe for a line $\hat{l} = (a,b,c)$, the vector (a,b) is normal to $\tilde{l}$.

Normalize $\tilde{l} = (\hat{n}_x, \hat{n}_y, d)$, such that $\|\hat{n}\| = 1$, and d is the distance to the origin

It is convenient to represent $\hat{n}$ using angle ϕ

line 1 Slope $\frac{-3}{1}$, thus angle



, inv $\tan\left(\frac{3}{1}\right) = \phi \approx 1.25$

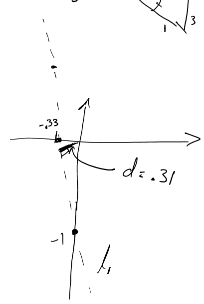
$\hat{n} = (\cos\phi, \sin\phi)$

thus $\hat{n} \approx (.95, .31)$, for line 1.

$\therefore\ l_1 = (.95, .31, .31)$

$= (\hat{n}_x, \hat{n}_y, d)$

distance to origin



4) Assume 2 lines $\hat{l}_1$ and $\hat{l}_2$, in 2D. Assume A is 2×3 matrix. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

Let $l_1 = (n_x, n_y, d_1)$ & $l_2 = (n_x, n_y, d_2)$

Let $Al_1 = l_1'$ & $Al_2 = l_2'$. Show if l_1 & l_2 are parallel, then l_1' & l_2' are parallel.

Thus

$l' \ [$

Thus

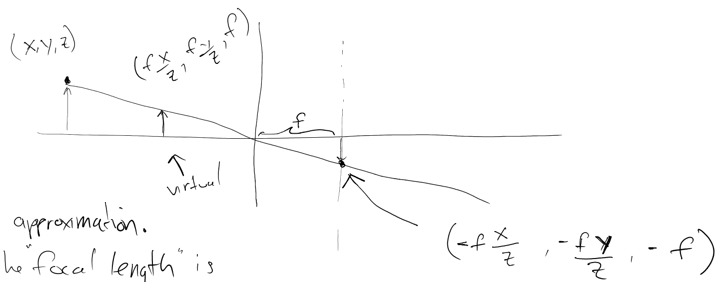
$$l_1' = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ d_1 \end{bmatrix} = \begin{bmatrix} a_{11}n_x + a_{12}n_y + a_{13}d_1 \\ a_{21}n_x + a_{22}n_y + a_{23}d_1 \end{bmatrix}$$

$$l_2' = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ d_2 \end{bmatrix} = \begin{bmatrix} a_{11}n_x + a_{12}n_y + a_{13}d_2 \\ a_{21}n_x + a_{22}n_y + a_{23}d_2 \end{bmatrix}$$

Observe the normal vectors are similar save a translation $\begin{bmatrix} a_{13}d_1 \\ a_{23}d_1 \end{bmatrix}$ or $\begin{bmatrix} a_{13}d_2 \\ a_{23}d_2 \end{bmatrix}$.

Thus the resulting lines are parallel.

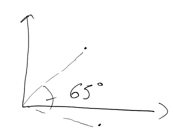
5)



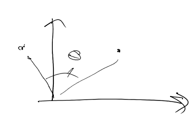
Here we use the pinhole approximation.
In this model the "focal length" is simply the distance from the optical center to the image plane.

5 A) $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $\quad \theta = -65°$

B) $R\vec{x} = R\begin{bmatrix} 4 \\ 5 \end{bmatrix} \cong \begin{bmatrix} 6.23 \\ -1.54 \end{bmatrix}$



or



6) $P = \underset{\text{intrinsic parameters}}{K} \underbrace{[R|t]}_{\text{extrinsic parameters}}$

R: A 3d rotation matrix with 9 entries. This can be parameterized with 3 angles of rotation across the 3 cardinal axes, or using 1 angle and 1 vector (to be rotated about).

t: 3x1 translation vector.

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$$K = \begin{bmatrix} f & s & c_x \\ & \alpha f & c_y \\ & & 1 \end{bmatrix}$$

Using pinhole approximation & using virtual plane.

f: focal length or distance to image plane from optical center

s: skew due to imperfections in camera.

c_x c_y: translation to camera coords.