COSC579: Active Contours: Snakes

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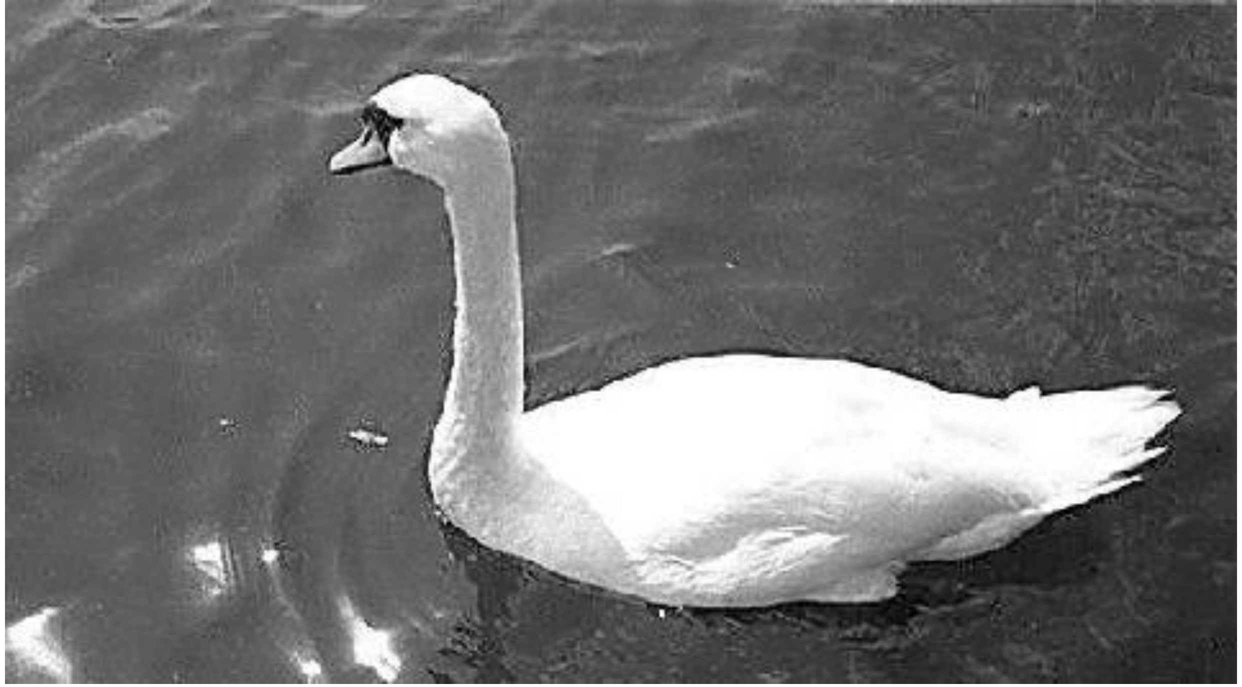
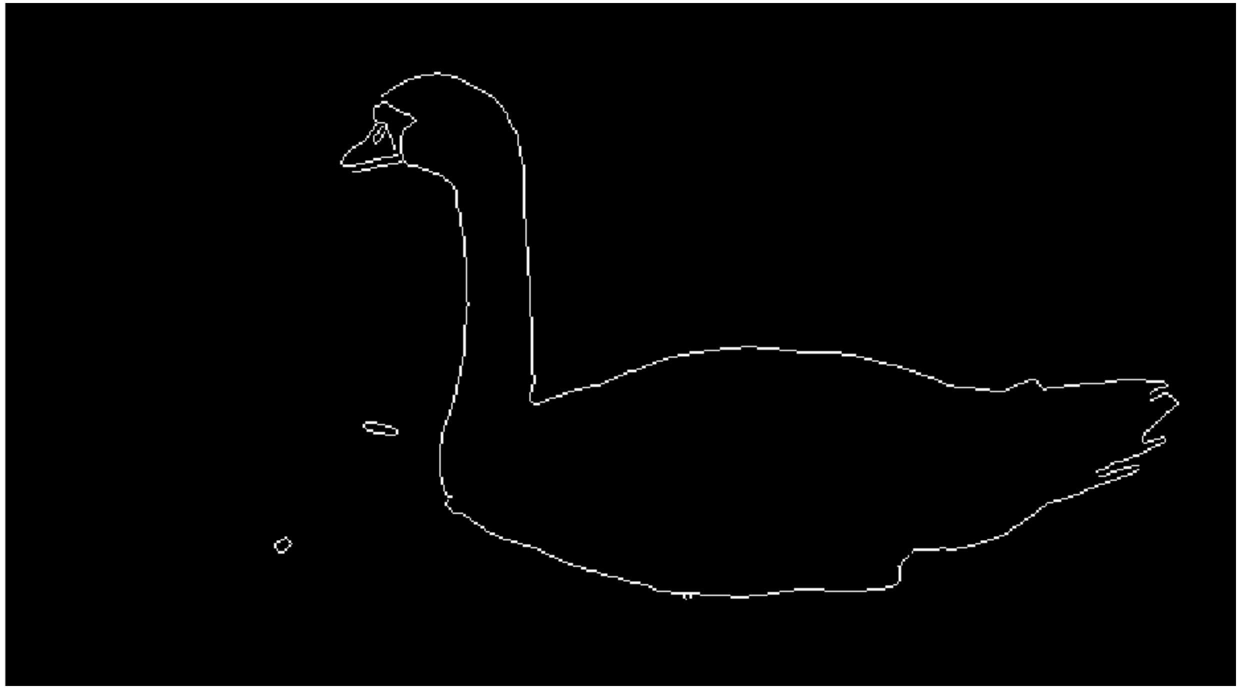
Outline

I. Snakes and Active Contours
    I. Find object boundaries using contours
    II. Energy Minimization using Variational Methods
II. Snake improvements: avoiding local optima
    I. Scale Space
        I. Examples in Radar
    II. Gradient Vector Flow (GVF Snakes)

Using edges and boundaries to guide segmentation

- Active Contours
- Snakes



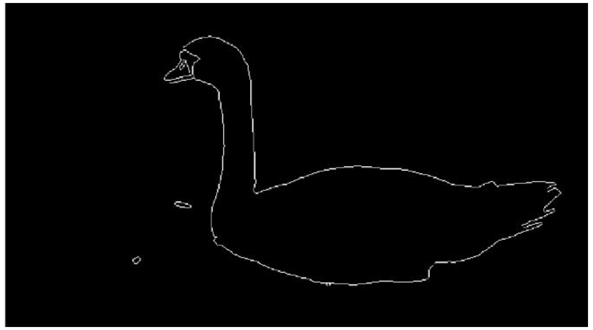


Sometimes edge detectors find the boundary pretty well.

Sometimes it’s not enough.

Modeling segment boundary

- Use a parametric contour to model segment boundary
- Using interpolation techniques to estimate boundary where edge information is lacking.
- ACTIVE contours
  - Use image feature to automatically choose boundary pixels
  - Based on (potential) energy functional



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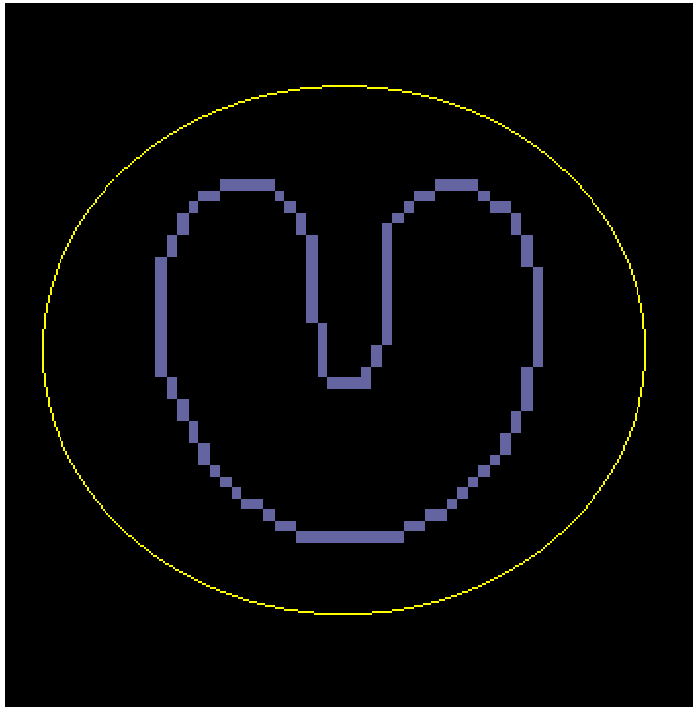
Framework for snakes

- A higher level process or a user initializes any curve *close to the object boundary.*
- The snake then starts *deforming* and moving towards the desired object boundary.
- In the end it completely “shrink-wraps” around the object.



(Diagram courtesy “Snakes, shapes, gradient vector flow”, Xu, Prince)

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Active Contours: Boundary Based Segmentation

- First introduced in 1987 by Kass et al, and gained popularity since then.
- Represents an object boundary or some other salient image feature as a **parametric curve.**
- Goal: Find "Best" Contour
  - An energy functional E is associated with the curve.
  - The problem of finding object boundary is cast as an energy minimization problem.
  - Key: Derive Energy terms that characterize the boundaries of objects in image data

Modeling

- The contour is defined in the (x, y) plane of an image as a parametric curve (Spline)

$$\mathbf{v}(s)=(x(s), y(s))$$

- Contour is said to **possess an energy** (E_{snake}) which can be defined as the sum of the three energy terms.

$$E_{snake} = E_{internal} + E_{external} + E_{constraint}$$

- The energy terms are defined ***cleverly*** in a way such that the final position of the contour will have a minimum energy (E_{min}) at desired boundary

- Therefore our problem of detecting objects reduces to an energy minimization problem.

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Internal Energy (E_{int})

- Depends on the intrinsic properties of the curve.
- Sum of elastic energy and bending energy.

Elastic Energy ($E_{elastic}$):

- The curve is treated as an elastic rubber band possessing elastic potential energy.
- It discourages stretching by introducing tension.

$$E_{elastic} = \frac{1}{2}\int_{s} \alpha(s)\,|v_s|^2\,ds \qquad v_s = \frac{dv(s)}{ds}$$

- Weight $\alpha(s)$ allows us to control elastic energy along different parts of the contour. Considered to be constant α for many applications.
- **Responsible for shrinking of the contour.

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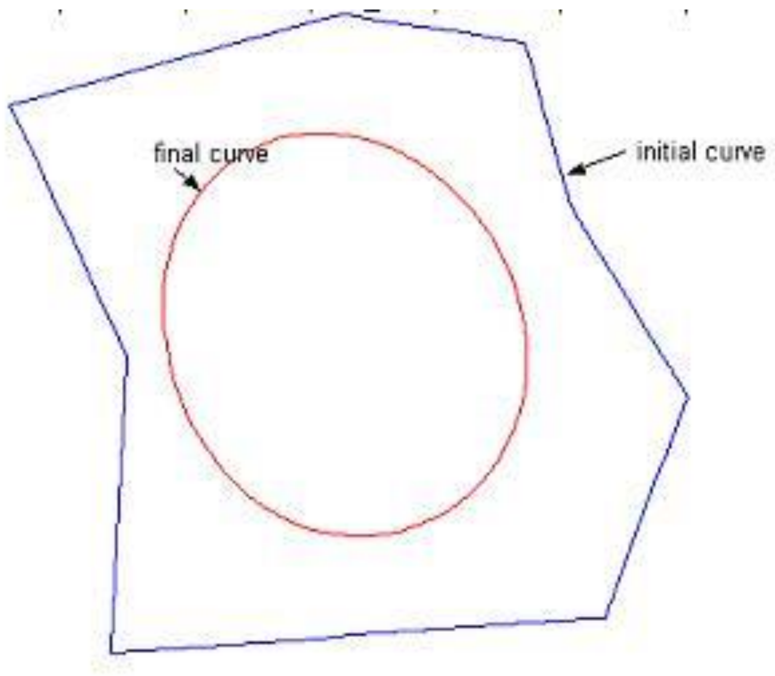
Elastic force

- Generated by elastic potential energy of the curve.

$$F_{elastic} = \alpha v_{ss}$$

- Characteristics (refer diagram)



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Bending Energy ($E_{bending}$):

- The snake is also considered to behave like a thin metal strip giving rise to bending energy.

- It is defined as sum of squared curvature of the contour.

$$E_{bending} = \frac{1}{2}\int_s \beta(s)\,|v_{ss}|^2\, ds$$

- **β(s)** plays a similar role to **α(s).**
- Bending energy is minimum for a circle.

- Total internal energy of the snake can be defined as

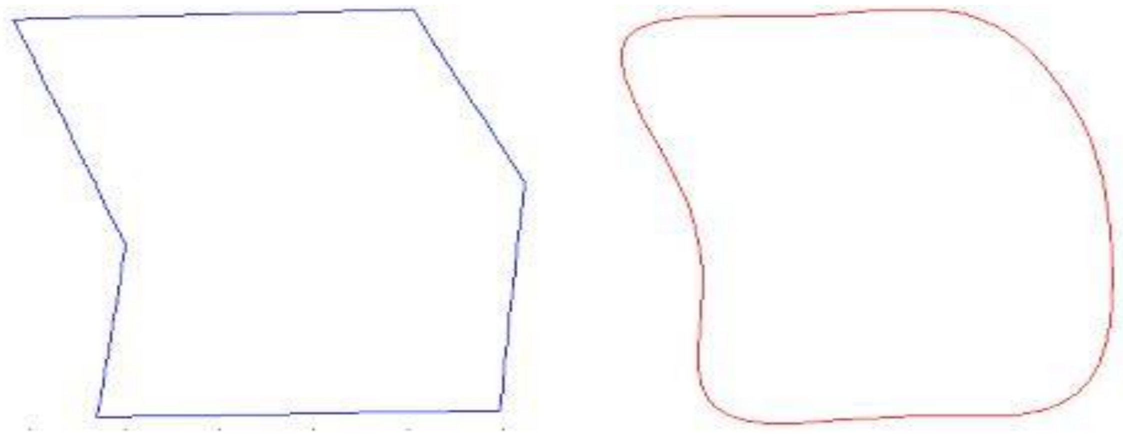
$$E_{\text{int}} = E_{elastic} + E_{bending} = \int_s \frac{1}{2}(\alpha\,|v_s|^2 + \beta\,|v_{ss}|^2)ds$$

Using finite differences …

$$E_{\text{int}}(i) = \alpha_i|\mathbf{v}_i - \mathbf{v}_{i-1}|^2/2h^2 + \beta_i|\mathbf{v}_{i-1} - 2\mathbf{v}_i + \mathbf{v}_{i+1}|^2/2h^4$$

Bending force

- Generated by the bending energy of the contour.
- Characteristics (refer diagram):



Initial curve
(High bending energy)

Final curve deformed by bending force. (low bending energy)

- Thus the bending energy tries to smooth out the curve.

External energy of the contour (E_{ext})

- It is derived from the image.
- Define a function $E_{image}(x,y)$ so that **it takes on its smaller values at** the features of interest, such as **boundaries.**

$$E_{ext} = \int_s E_{image}(v(s))ds$$

Key rests on defining $E_{image}(x,y)$. Some examples

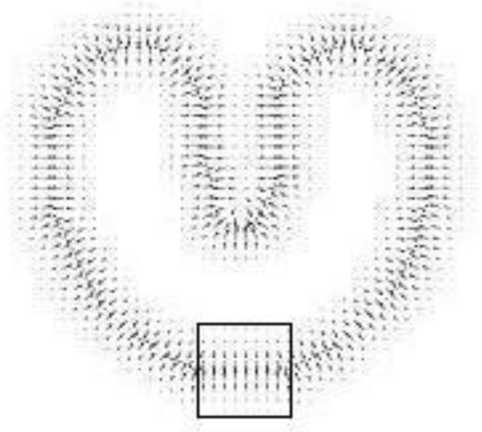
- $E_{image}(x,y) = -|\nabla \mathrm{I}(x,y)|^2$
- $E_{image}(x,y) = -|\nabla(G_\sigma(x,y) * I(x,y))|^2$

External force

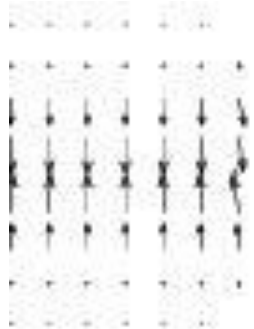
$$F_{ext} = -\nabla E_{image}$$

- It acts in the direction so as to minimize E_{ext}

Image



External force



Zoomed in

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Energy and force equations

- The problem at hand is to find a contour v(s) that minimize the energy functional

$$E_{snake} = \int_s \frac{1}{2}(\alpha(s)|v_s|^2 + \beta(s)|v_{ss}|^2) + E_{image}(v(s))ds$$

- Using variational calculus and by applying Euler-Lagrange differential equation we get following equation

$$\alpha v_{ss} - \beta v_{ssss} - \nabla E_{image} = 0$$

- Equation can be interpreted as a force balance equation.

- Each term corresponds to a force produced by the respective energy terms. The contour deforms under the action of these forces.

Discretizing

- the contour v(s) is represented by a set of control points

  $v_0, v_1, \ldots\ldots, v_{n-1}$

- The curve is piecewise linear obtained by joining each control point.
- Force equations applied to each control point separately.
- Each control point allowed to move freely under the. influence of the forces.
- The energy and force terms are converted to discrete form with the derivatives substituted by finite differences.

Euler Equation using finite differences

$$
\begin{aligned}
\alpha_i(\mathbf{v}_i - \mathbf{v}_{i-1}) - \alpha_{i+1}(\mathbf{v}_{i+1} - \mathbf{v}_i)& \\
+ \beta_{i-1}[\mathbf{v}_{i-2} - 2\mathbf{v}_{i-1} + \mathbf{v}_i]& \\
- 2\beta_i[\mathbf{v}_{i-1} - 2\mathbf{v}_i + \mathbf{v}_{i+1}]& \\
+ \beta_{i+1}[\mathbf{v}_i - 2\mathbf{v}_{i+1} + \mathbf{v}_{i+2}]& \\
+ (f_x(i), f_y(i)) = 0&
\end{aligned}
$$

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- Consider the snake to also be a function of time i.e. $v_t(s,t)$

$$\alpha v_{ss}(s,t) - \beta v_{ssss}(s,t) - \nabla E_{image} = v_t(s,t) \qquad v_t(s,t) = \frac{\partial v(s,t)}{\partial t}$$

- If RHS=0 we have reached the solution.

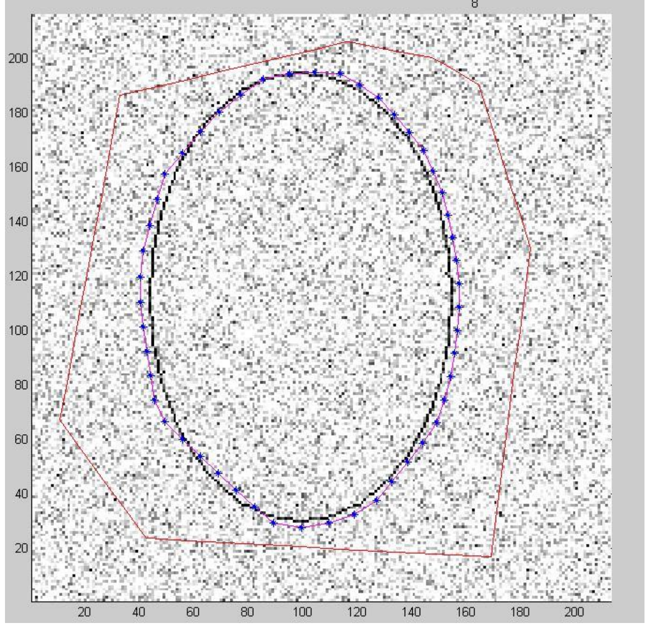
- On every iteration update control point only if new position has a lower external energy.

- Snakes are very sensitive to local minima which leads to wrong convergence.

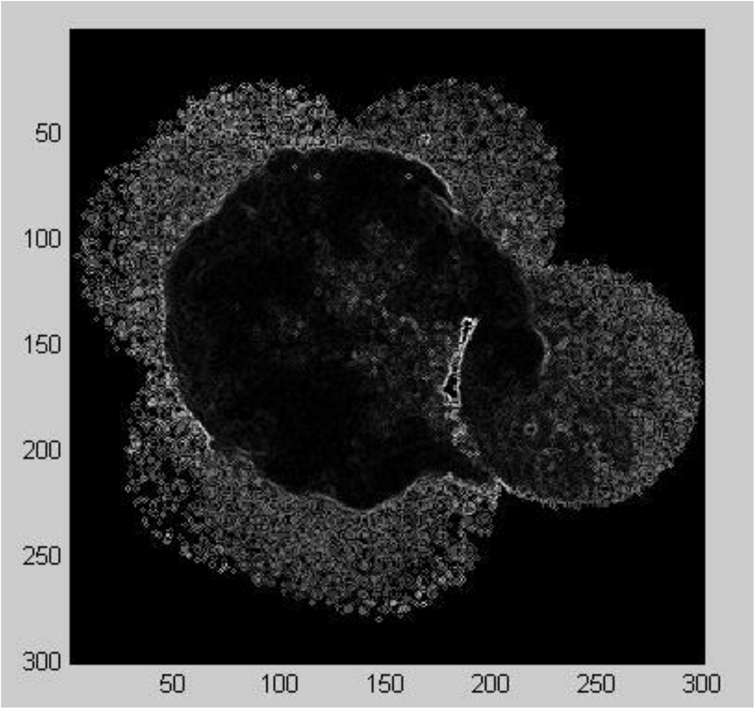
$$\mathbf{A}\mathbf{x}_t + \mathbf{f}_\mathbf{x}(\mathbf{x}_{t-1}, \mathbf{y}_{t-1}) = -\gamma(\mathbf{x}_t - \mathbf{x}_{t-1})$$
$$\mathbf{A}\mathbf{y}_t + \mathbf{f}_\mathbf{y}(\mathbf{x}_{t-1}, \mathbf{y}_{t-1}) = -\gamma(\mathbf{y}_t - \mathbf{y}_{t-1})$$

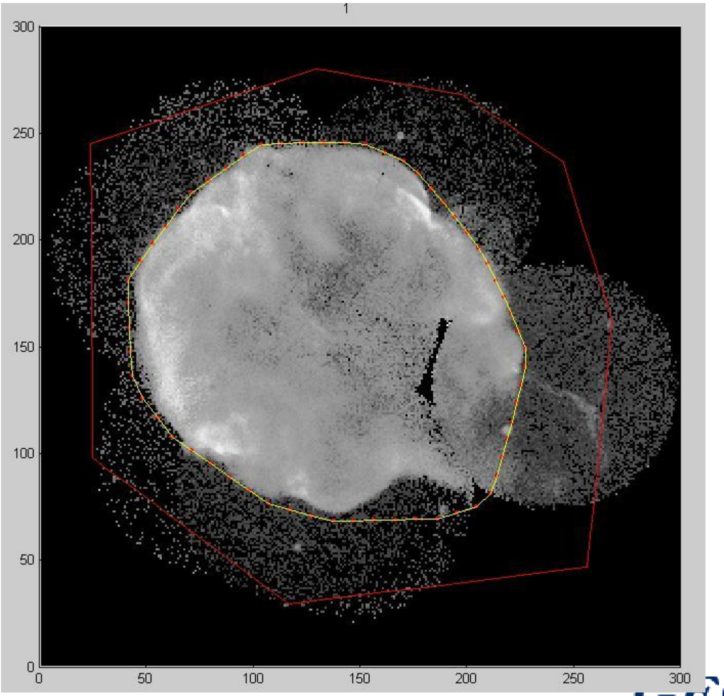
Corresponding Update Equations (A contains coeffs for internal forces)

$$\mathbf{x}_t = (\mathbf{A} + \gamma\mathbf{I})^{-1}(\gamma\mathbf{x}_{t-1} - \mathbf{f}_\mathbf{x}(x_{t-1}, y_{t-1}))$$
$$\mathbf{y}_t = (\mathbf{A} + \gamma\mathbf{I})^{-1}(\gamma\mathbf{y}_{t-1} - \mathbf{f}_\mathbf{y}(x_{t-1}, y_{t-1}))$$



- Noisy image with many local minimas
- WGN sigma=0.1
- Threshold=15





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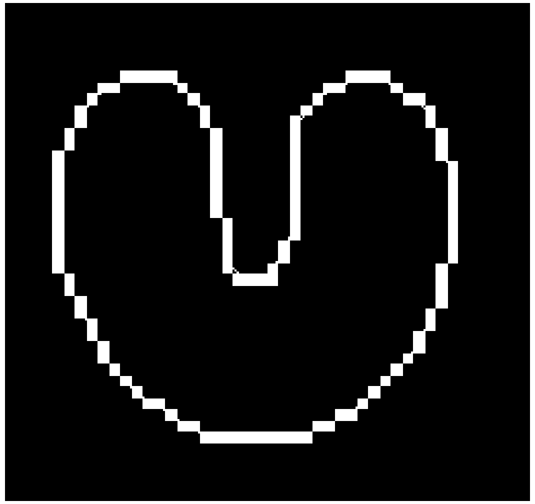
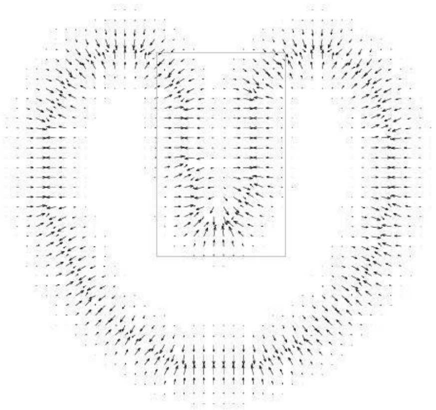
Weakness of traditional snakes (Kass model)

- Extremely sensitive to parameters.
- Small capture range.





- No external force acts on points which are far away from the boundary.
- Convergence is dependent on initial position.

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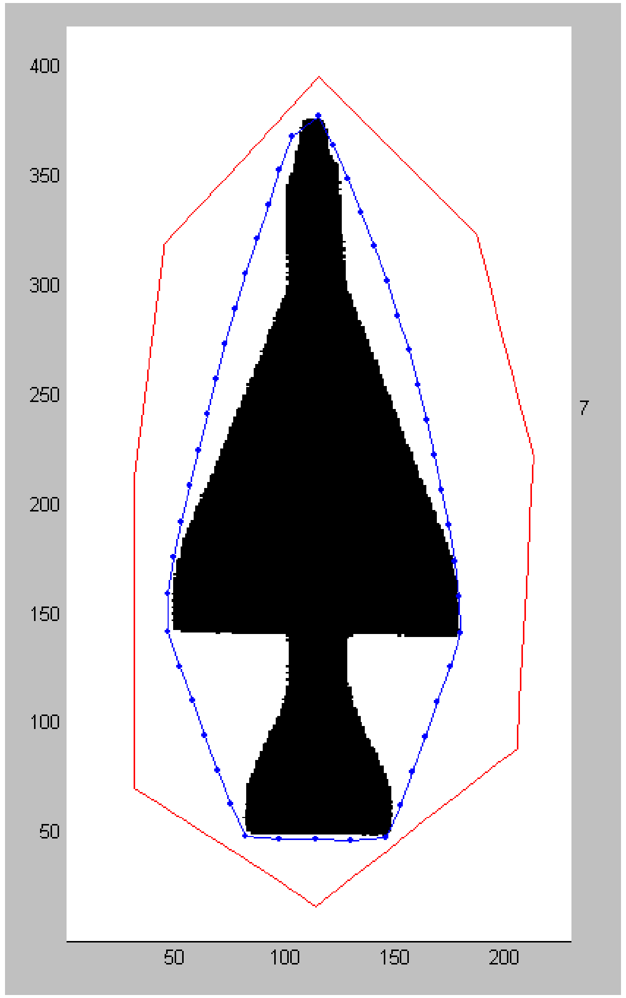
Weakness (contd…)

- Fails to detect concave boundaries. External force cant pull control points into boundary concavity.



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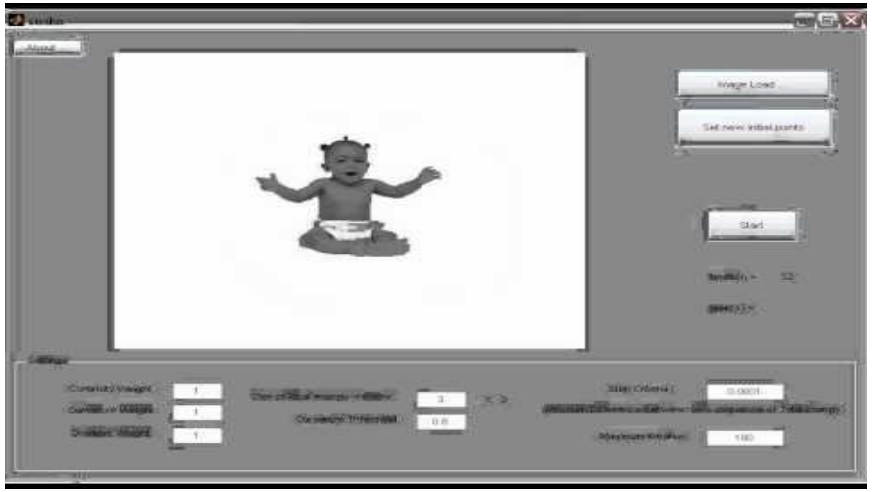
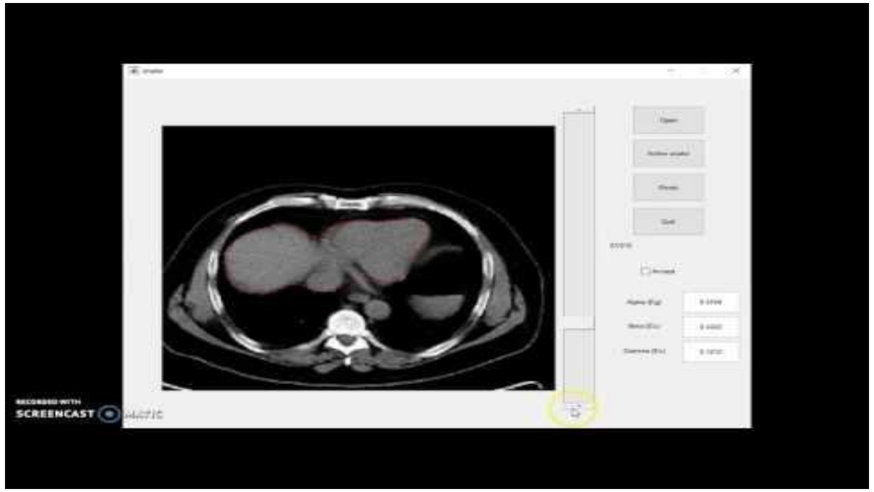
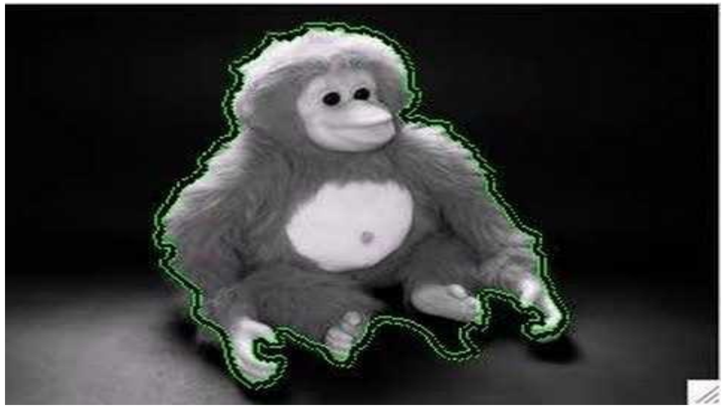
Other Examples







Avoiding Local Minima

- Two approaches discussed here
  - Scale Space
  - Gradient Vector Flow

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Scale Space to avoid local optima

Smooth image features with a large sigma
Slowly (iteratively) reduce sigma

In order to show the relationship of scale-space continuation to the Marr–Hildreth theory of edge-detection [10], we have experimented with a slightly different edge functional. The edge-energy functional is

$$E_{\text{line}} = -(G_\sigma * \nabla^2 I)^2 \qquad (5)$$

- Add energy term to snake means that it is attracted to zero-crossings but still constrained by its own smoothness.

See Radar Examples

Gradient Vector Flow (GVF)

(A new external force for snakes)

- Detects shapes with boundary concavities.
- Large capture range.

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Model for GVF snake

- The GVF field is defined to be a vector field
  V(x,y) = $(u(x,y), v(x,y))$
- Force equation of GVF snake (with new ext term V)

$$\alpha v_{ss} - \beta v_{ssss} + V = 0$$

- V(x,y) is defined such that it minimizes the energy functional

$$E = \iint \mu(u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |V - \nabla f|^2 \, dxdy$$

f(x,y) is the edge map of the image.

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- GVF field can be obtained by solving following equations

$$\mu\nabla^2 u-(u-f_x)(f_x^2+f_y^2)=0$$

$$\mu\nabla^2 v-(v-f_y)(f_x^2+f_y^2)=0$$

∇^2 Is the Laplacian operator.

- Reason for detecting boundary concavities.

- The above equations are solved iteratively using time derivative of u and v.

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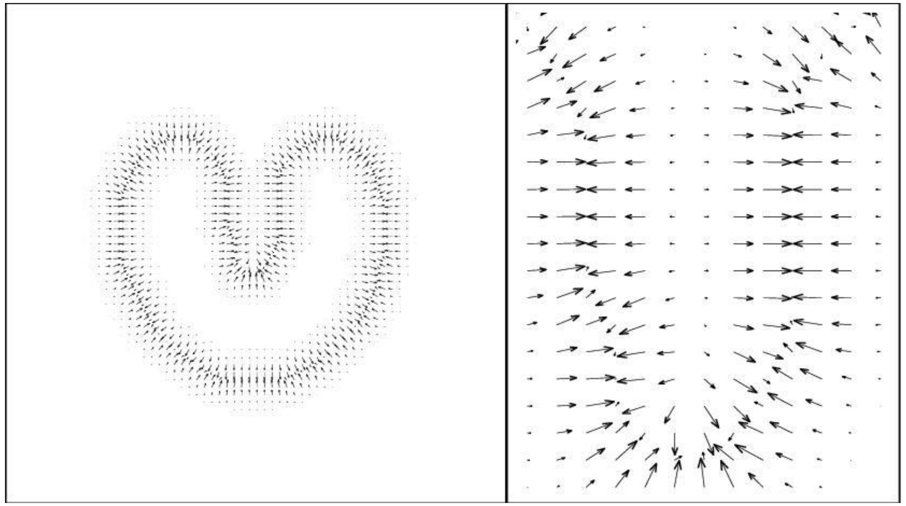
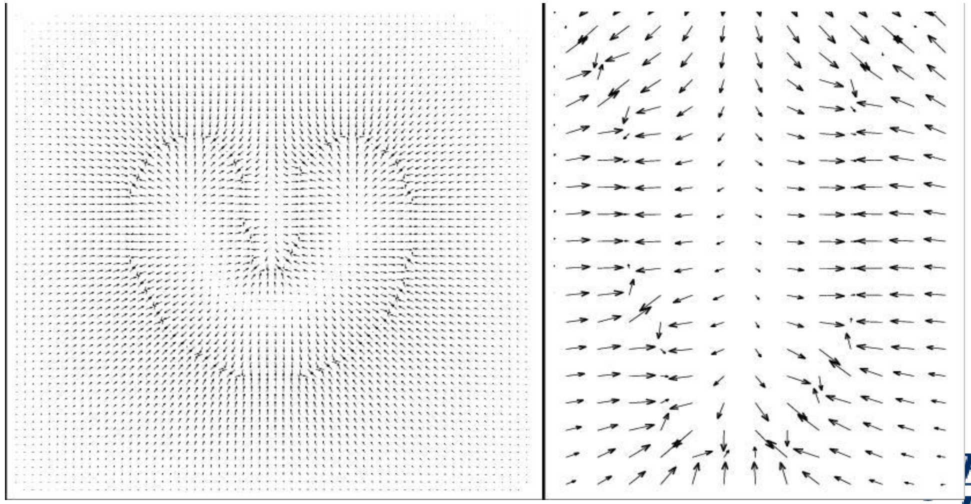
Traditional external force field v/s GVF field

Traditional force



GVF force



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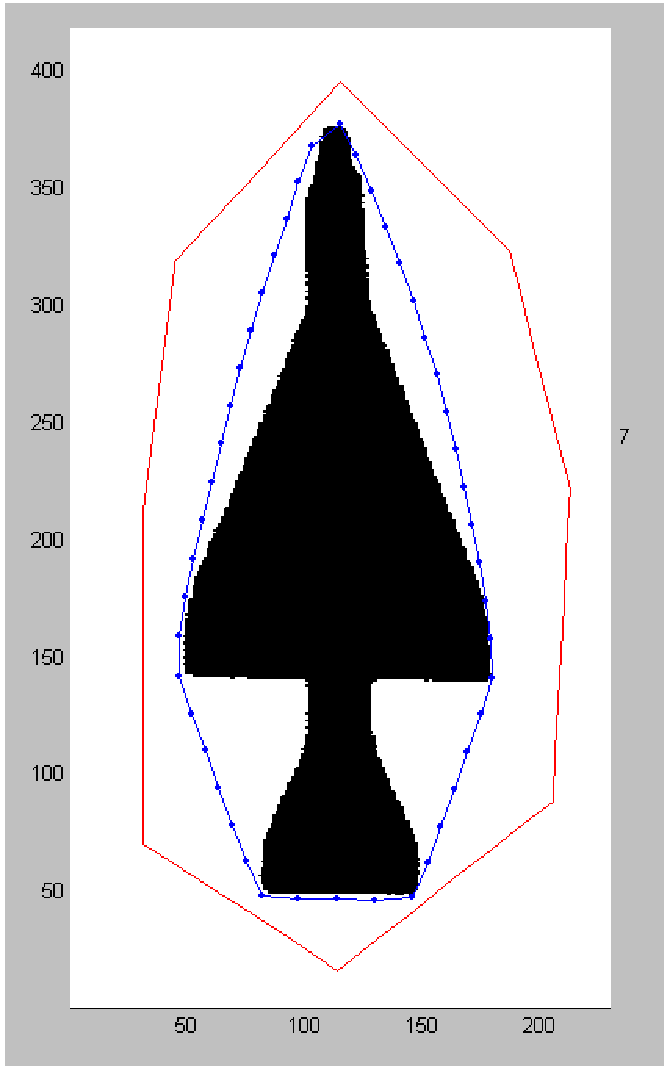
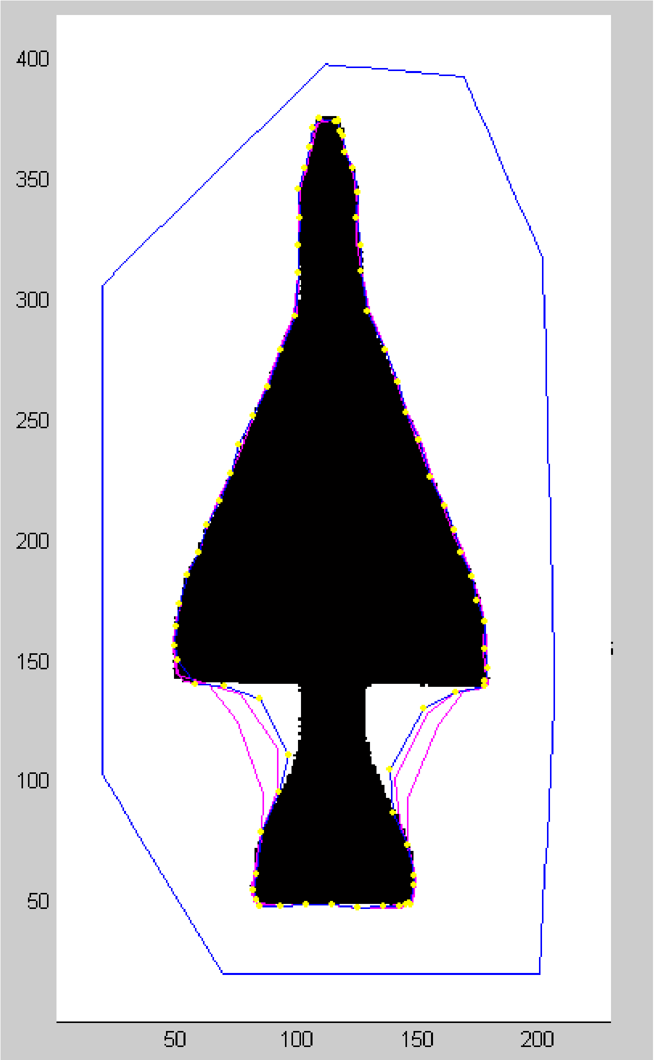
(Diagrams courtesy “Snakes, shapes, gradient vector flow”, Xu, Prince)

Results



Traditional snake



GVF snake

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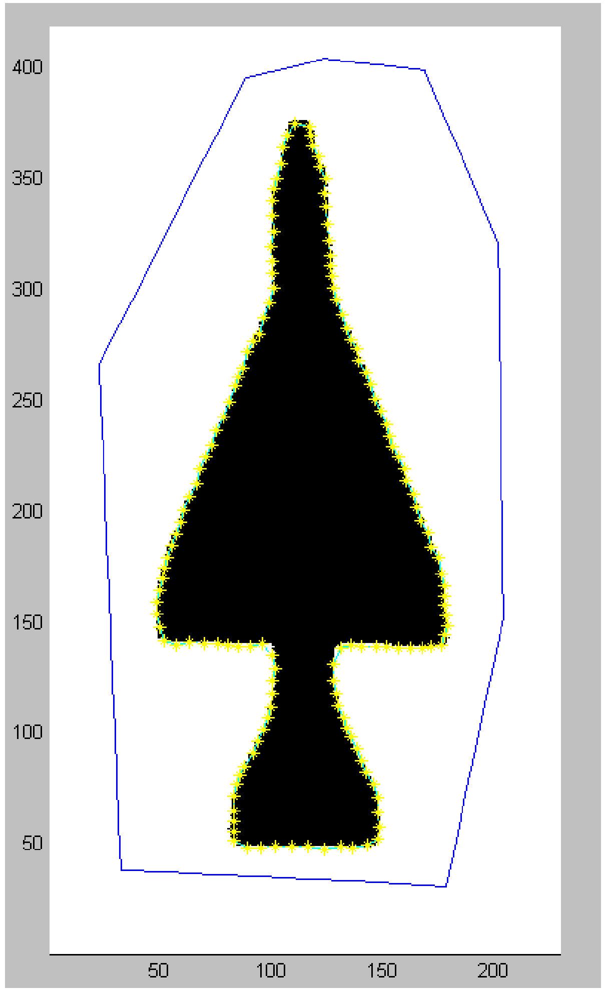
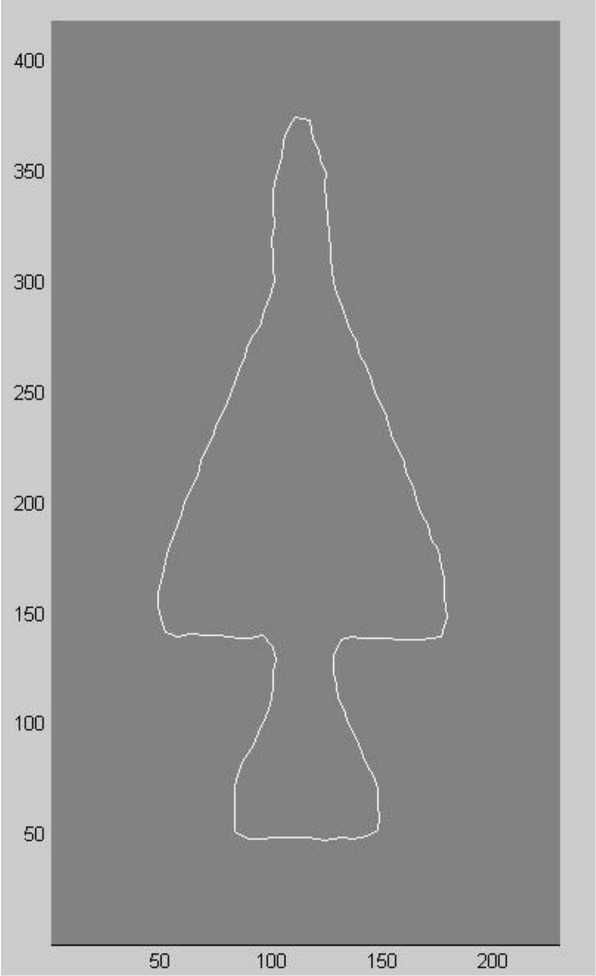
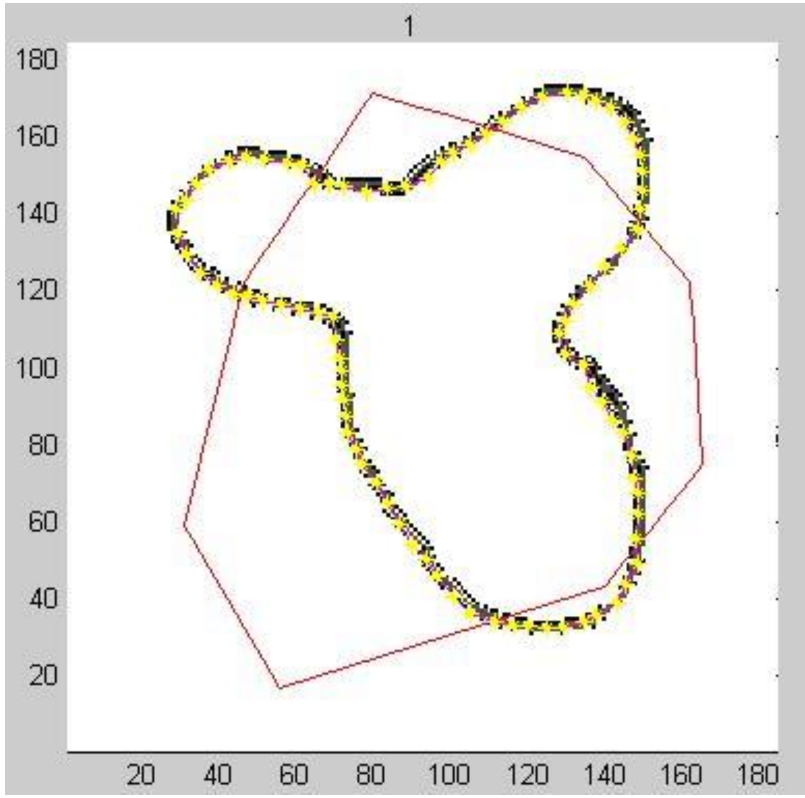
Cluster and reparametrize the contour dynamically.

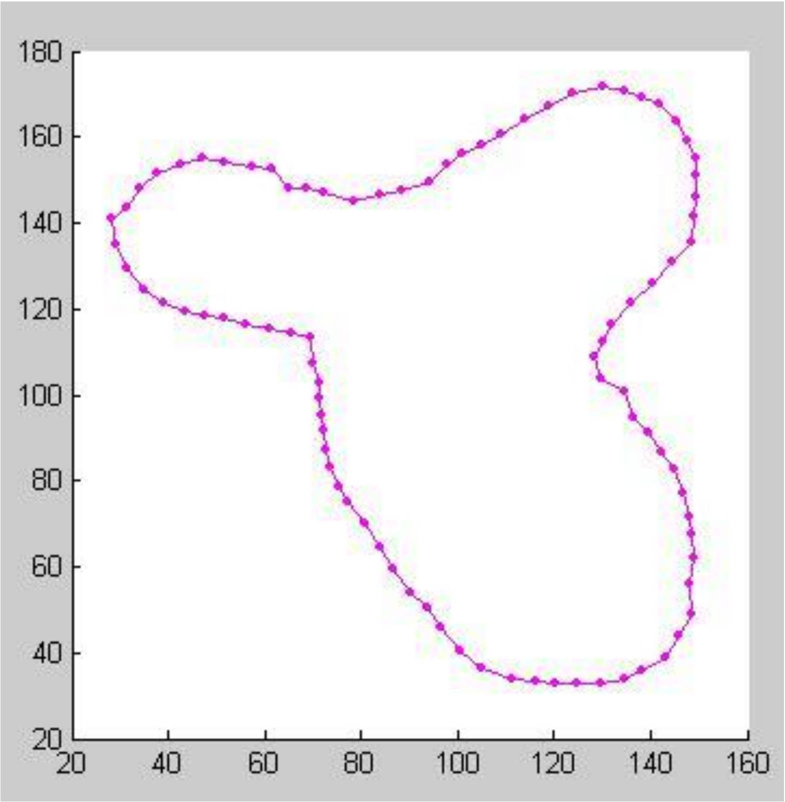




Final shape detected

The contour can also be initialized across the boundary of object!!
Something not possible with traditional snakes.





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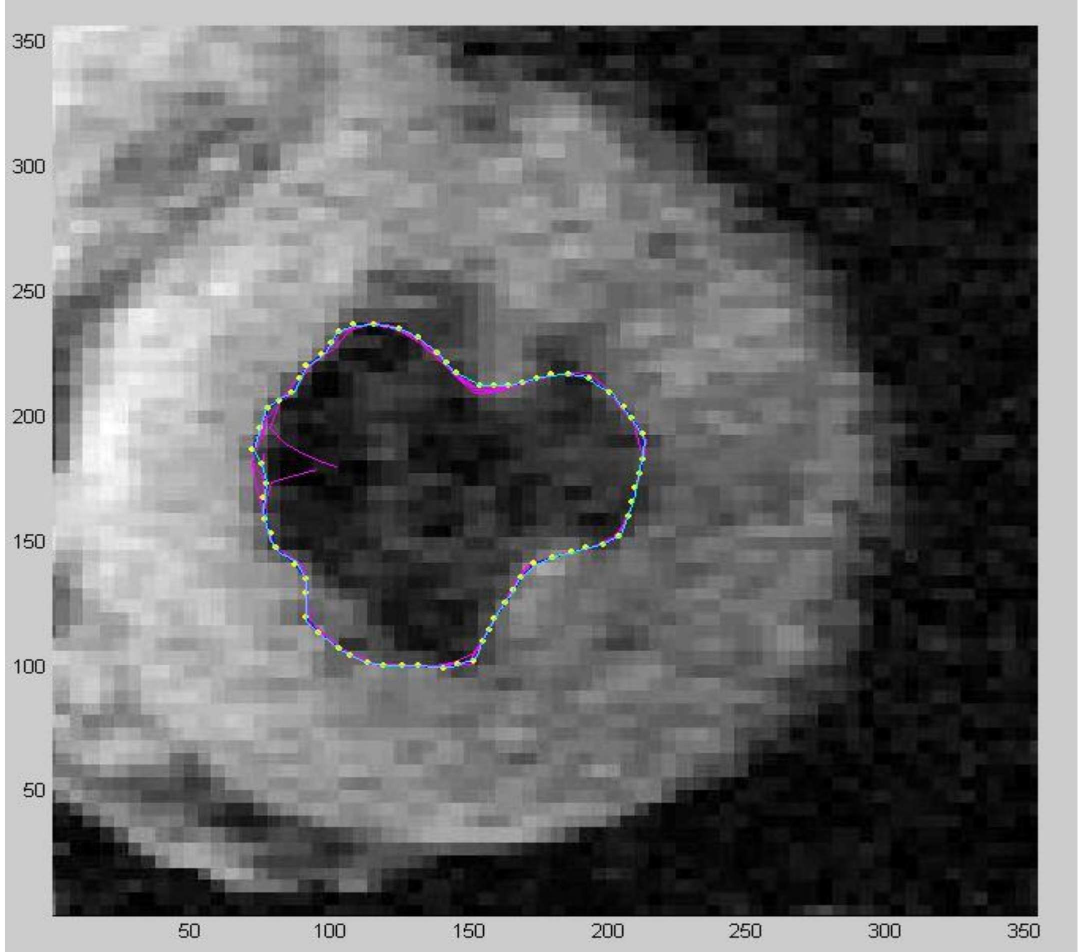
Medical Imaging



Magnetic resonance image of the left ventricle of human heart

Notice that the image is poor quality with sampling artifacts

Problem with GVF snake

- Very sensitive to parameters.
  - As are most similar methods
- Slow. Finding GVF field is computationally expensive.

Applications of snakes

- Image segmentation particularly medical imaging community (tremendous help).
- Motion tracking.
- Stereo matching.
- Shape recognition.

References

- M. Kass, A. Witkin, and D. Terzopoulos, "Snakes: Active contour models.“, International Journal of Computer Vision. v. 1, n. 4, pp. 321-331, 1987.

- Chenyang Xu and Jerry L. Prince , "Snakes, Shape, and Gradient Vector Flow“, IEEE Transactions on Image Processing, 1998.

- C. Xu and J.L. Prince, “Gradient Vector Flow: A New External Force for Snakes”, Proc. IEEE Conf. on Comp. Vis. Patt. Recog. (CVPR), Los Alamitos: Comp. Soc. Press, pp. 66-71, June 1997.

COSC160: Appendix

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